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Offset of rotation centers creates a bias in isokinetics: A virtual model including stiffness or friction

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Abstract

The present paper deals with a virtual model devoted to isokinetics and isometrics assessment of a human muscular group in the common joints, knee, ankle, hip, shoulder, cervical spine, etc. This virtual model with an analytical analysis followed by a numerical simulation is able to predict measurement errors of the joint torque due to offset of rotation centers between the body segment and the ergometer arm. As soon as offset is present, errors increase due to the influence of inertial effects, gravity effects, stiffness due to the limb strapping on the ergometer arm or Coulomb friction between limb and ergometer. The analytical model is written in terms of Lagrange formalism and the numerical model uses ADAMS software adapted to multi-body dynamics simulations. Results of models show a maximal relative error of 11%, for a 10% relative offset between the rotation centers. Inertial contributions are found to be negligible but gravity effects must be discussed in regard to the measured torque. Stiffness or friction effects may also increase the torque error; in particular when offset occurs, it is shown that errors due to friction have to be considered for all torque level while only stiffness effects have to be considered for torque less than 25 N m. This study also emphasizes the influence of the angular range of motion at a given angular position.

Keywords: Isokinetics; Isometrics; Assessment; Stiffness; Friction

1. Introduction

Kinematics of human joints is essential in orthopaedics, rehabilitation medicine (Woltring et al., 1985) and also in biomechanical motion analysis (Chang and Pollard, 2006; Halvorsen et al., 1999). This analysis must be completed by joint forces and direct torque measurements (Begon et al., 2006). Thus the need for specific mechanical apparatus is unavoidable. A single arm ergometer can be used for such purposes; it gives access to the two main types of exercises: isokinetics and isometrics. In both cases, the ergometer must deliver force or torque value with absolute precision.

An isokinetic exercise is based on a forced movement of a limb with constant angular velocity. Whether the muscle group acting on the limb drives the ergometer (concentric contraction) or is driven by the ergometer (eccentric contraction) is of no importance. The constant velocity is obtained by a digitally controlled servo-motor. Isometric or isokinetic measurements on joints such as knee (Manou et al., 2002), ankle (Gleeson and Mercer, 1992), or cervical spine (Portero and Genriès, 2003; Olivier and Du Toit, 2008) are used for the assessment of muscular capacity or muscular fatigue. However, for isometrics or isokinetics, authors strongly insist on influence of experimental clinical procedures such as subject position (Falkel et al., 1987; Hageman et al., 1989; Miller et al., 1997), gravity (Winter et al., 1981), inter-experimentalist, inter-ergometer, inter-test or inter-subject reproducibility (Mayer et al., 1994), inertial effects (Lossifidou and Baltzopoulos, 1998) and also alignment of joint and ergometer axis (Sorensen et al., 1998; Rothstein et al., 1987). This last point is generally underlined and is the object of a repeated recommendation. To our knowledge, only one theoretical study (Reimann et al., 1997) was published earlier though

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Nomenclature

C_e	torque measured by the ergometer
C_m	torque produced by the muscular group at the joint
T	ratio between measured and muscle torque or correction factor ($T = -C_e/C_m$)
α	rotation angle of the ergometer arm
α_0	initial rotation angle of the ergometer arm
$\dot{\alpha}$	angular velocity of ergometer arm (constant)
β	rotation angle of the limb
$\dot{\beta}$	angular velocity of limb (variable)
V	ratio between limb and ergometer arm velocity ($V = \dot{\beta}/\dot{\alpha}$)
d	offset distance between ergometer rotation center and limb rotation center
φ	offset angular position
G_m	limb center of mass
G_e	ergometer arm center of mass
l_{Gm}	distance between joint rotation center and limb center of mass
l_{Ge}	distance between joint rotation center and ergometer arm center of mass
l_m	length of the limb (variable)
l_e	length of the ergometer arm (constant)
I_m	limb inertia (proximal)
I_e	ergometer arm inertia (with respect to rotation axis)
m_m	limb mass
m_e	ergometer arm mass
E	relative error defined by $E = d/l_e$
f	Coulomb friction coefficient
k	stiffness due to strapping
c	viscous damping associated to stiffness
O_e	ergometer rotation center
O_m	joint rotation center
ROM	range of motion
$\alpha_{P,}$ $\alpha_{P1,}$ $\alpha_{P2,}$ α_{P3}	initial angular positions of the ergometer arm for different ROM

in fact this paper was based on observations and measurements. However, these authors presented the influence of rotation center offset on measured torque, angle and angular velocity. In particular, they showed that a 10% relative error of axis misalignment can lead to a 10% torque relative error.

In general, the articular joint axis and the ergometer axis may be situated anywhere. The torque component is measured in the direction of the ergometer axis. The misalignment of axes is then reduced to an offset between rotation centers of limb and rotation center of ergometer. In many types of joints, this offset varies in distance and direction, the evaluation of which is not practicable. Therefore, the hypothesis of a constant offset is proposed here. The goal of the present paper is to show with a virtual model that the main source of measurement errors comes from this offset. Only when offset occurs do additional errors originate from other parameters: inertial effects, gravity effects, strapping stiffness mainly originating from flesh movements on the limb bone or friction between the limb and the ergometer arm. These last two effects are considered as mutually exclusive.

In practice, it may be difficult to reduce the offset to zero. Moreover, it happens that the rotation center of the joint moves during the exercise, the knee for example, thus preventing the offset zeroing. Anyhow, in the case of variable rotation center for the limb, limb movement is necessary otherwise the limb may be damaged.

The interest of the paper is to propose methods to identify correction factors taken into account the different relevant parameters such as inertia, gravity, strapping stiffness or friction between limb and ergometer arm in the presence of an offset. This model could also be used to alert isokinetics users to the importance of considering the relevant parameters. If relative results are expected, raw data are sufficient, but, for asolute and accurate results, a correction factor must be introduced.

2. Methods

2.1. Analytical model of offset

The proposed virtual model can be applied to any human joint. In Fig. 1, we show two examples of ergometer arm and limb misalignment for the elbow or the knee. In this paper, the parametric description of the model is based on the knee. Fig. 2 shows the mechanical variables and parameters necessary to write the mechanics equations that permit the expression of the ratio T between the ergometer measured torque C_e and the torque C_m produced by the joint muscles. In the paper, this ratio is also known as torque correction factor.

In the Lagrange formalism, we express kinetic energy E_c, potential energy E_p restricted to gravity and power P of efforts,

$$E_c = \tfrac{1}{2} I_m \dot{\beta}^2 + \tfrac{1}{2} I_e \dot{\alpha}^2 \tag{1}$$

$$E_p = -m_m g (l_{Gm} \sin\beta + d \sin\varphi) - m_e g l_{Ge} \sin\alpha \tag{2}$$

$$P = C_e \dot{\alpha} + C_m \dot{\beta} \tag{3}$$

The angles α and β are not independent. At the velocity level their dependence is written through a linear relationship,

$$V\dot{\alpha} - \dot{\beta} = 0 \tag{4}$$

which can be written as

$$A\dot{q} = 0 \quad \text{with } \dot{q} = \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix}, \; A = \begin{bmatrix} V & -1 \end{bmatrix} \tag{5}$$

Since the generalized coordinates α and β are dependent, we introduce a Lagrange multiplier λ. Denoting the power coefficients by Q, the Lagrange equations are the following:

$$\frac{d}{dt}\frac{\partial E_c}{\partial \dot{q}} - \frac{\partial E_c}{\partial q} = Q - \frac{\partial E_p}{\partial q} + A^t \lambda \tag{6}$$

where q is the vector of generalized coordinates.

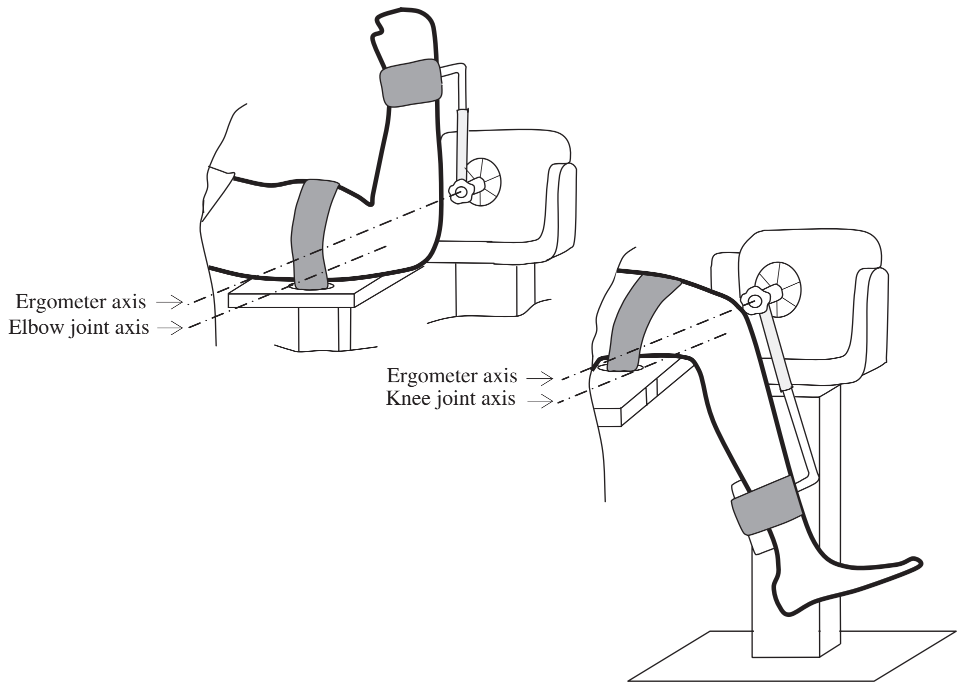

Fig. 1. Ergometer, elbow and leg fixation.

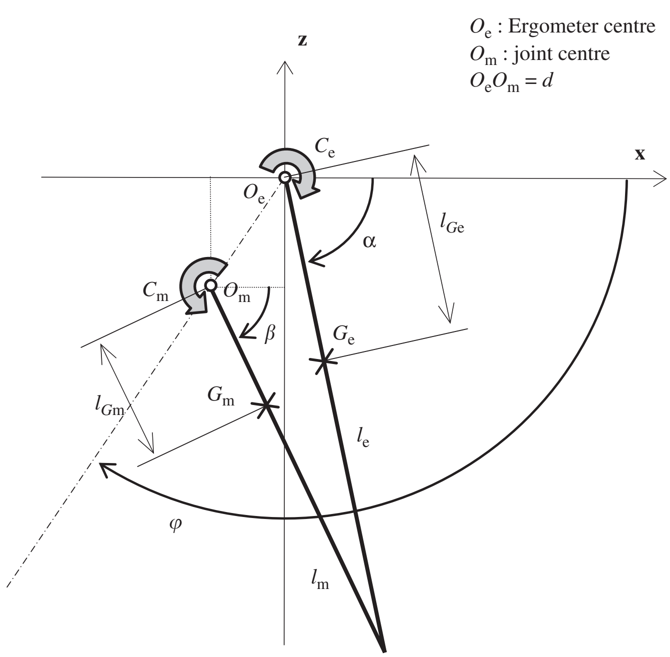

Fig. 2. Mechanical variables and parameters.

Eq. (6) leads to equations in α and β, and with the constrained Eq. (4), we have,

$$\begin{cases} I_e\ddot{\alpha} = C_e + V\lambda - \dfrac{\partial E_p}{\partial \alpha} \\ I_m\ddot{\beta} = C_m - \lambda \\ V\dot{\alpha} - \dot{\beta} = 0 \end{cases} \tag{7}$$

since the movement is only considered in the isokinetic phase, $\ddot{\alpha} = 0$. Referring to Fig. 2, angle β can be expressed as

$$\beta = \arctan\left(\frac{l_e \sin\alpha - d \sin\varphi}{l_e \cos\alpha - d\cos\varphi}\right) \tag{8}$$

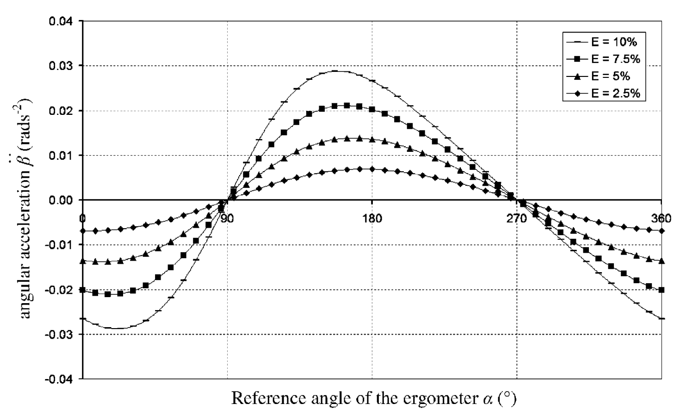

Fig. 3. Acceleration $\ddot{\beta}$ vs. α for $\varphi = \pi/2$ and $E(\%) = \{0, 2.5, 5, 7.5, 10\}$.

Replacing d by $d = El_e$,

$$\beta = \arctan\left(\frac{\sin\alpha - E\sin\varphi}{\cos\alpha - E\cos\varphi}\right) \tag{9}$$

In general, $E \ll 1$. Therefore, a first-order Taylor expansion with respect to E leads to

$$\beta \simeq \alpha + E\sin(\alpha - \varphi) \tag{10}$$

and to its time derivatives,

$$\dot{\beta} \simeq \dot{\alpha} + E\dot{\alpha}\cos(\alpha - \varphi) \tag{11}$$

$$\ddot{\beta} \simeq -E\dot{\alpha}^2 \sin(\alpha - \varphi) \tag{12}$$

Referring to Eq. (12) and Fig. 3, acceleration $\ddot{\beta}$ is found to have a maximum of 0.03 rad s^{-2} for a 10% relative offset error$E$. This variation is shown for $\varphi = \pi/2$ and $\dot{\alpha} = 30^\circ\,s^{-1}$. The corresponding dynamic torque is $I\ddot{\beta}$ where I_m is the proximal moment of inertia of the limb. Taking $I_m = 0.2\,kg\,m^2$ in order to overestimate limb inertia effects, we get a torque of 0.006 N m, which can be considered as negligible in comparison to the torques usually measured in isokinetics (see Table 1). If $\dot{\alpha}$ were equal to

$300^\circ\,\mathrm{s}^{-1}$, the inertial torque created would reach a maximum of 0.6 N m. In general, velocities are much lower and consequently inertia effects due to offset need not be considered. Then, system (7) reduces to the following equation:

$$C_\mathrm{m} = -\frac{C_\mathrm{e}}{V} + \frac{1}{V}\frac{\partial E_\mathrm{p}}{\partial \alpha} \tag{13}$$

with

$$\begin{aligned}\frac{1}{V}\frac{\partial E_\mathrm{p}}{\partial \boldsymbol{\alpha}} &= -\frac{m_\mathrm{e} g l_\mathrm{Ge}}{V}\cos\boldsymbol{\alpha} - \frac{m_\mathrm{m} g l_\mathrm{Gm}\cos\beta}{V}\left(\frac{\partial\beta}{\partial\alpha}\right)\\ &= -\frac{m_\mathrm{e} g l_\mathrm{Ge}}{V}\cos\alpha - m_\mathrm{m} g l_\mathrm{Gm}\cos\beta\end{aligned} \tag{14}$$

Table 1
Extreme torque amplitude in isokinetics

Joint	Minimum torque (N m)	Maximum torque (N m)
Hip (Dvir, 2003)	21	177
Knee (Dvir, 2003)	6.8	255
Ankle (Dvir, 2003)	12	183
Trunk (Dvir, 2003)	99.1	333.6
Shoulder (Dvir, 2003)	3.5	123.4
Elbow (Dvir, 2003)	13.5	62.2
Wrist (Dvir, 2003)	3	33.4
Cervical rachis (Portero and Genriès, 2003; Olivier and Du Toit, 2007)	6	69.4

In general, gravity corrections are directly included in modern ergometers taking into account the mass and center of mass position of the limb. However, the ergometer does not take the offset into consideration and a residual torque error ΔG remains,

$$\Delta G = m_\mathrm{m} g l_\mathrm{Gm}\left(\cos\beta - \frac{\cos\alpha}{V}\right) \simeq m_\mathrm{m} g l_\mathrm{Gm} E\cos(\varphi - 2\alpha) \tag{15}$$

with a maximum ΔG_max,

$$\Delta G_\mathrm{max} \simeq m_\mathrm{m} g l_\mathrm{Gm} E \tag{16}$$

For the knee, taking $m_\mathrm{m} = 4.5\,\mathrm{kg}$ and $l_\mathrm{Gm} = 0.2\,\mathrm{m}$, we obtain $\Delta G_\mathrm{max} = 0.9\,\mathrm{N\,m}$. Therefore, due to the higher torques measured, we can drop gravity errors.

Since inertia and gravity corrections are ignored, $T = V$ and T can be derived from Eq. (7) as

$$T = \frac{E\cos(\alpha - \varphi) - 1}{2E\cos(\alpha - \varphi) - 1 - E^2} \simeq 1 + E\cos(\varphi - \alpha) \tag{17}$$

which shows the torque correction factor T dependence in E, α and φ. This is illustrated in Fig. 4 in the particular case of $\varphi = \pi/2$ by the variation of T with respect to ergometer arm rotation α and different values of relative error E. It should be remarked that for the knee, $E = 10\%$ corresponds to an offset $d = 4\,\mathrm{cm}$ for a 40 cm arm length as cited by authors (Reimann et al., 1997).

For static measurements, isometrics in particular, Eq. (7) has to be transformed. For isometrics, the correction factor T is the ratio of differential angles showing the same relationship as previously between the torques,

$$C_\mathrm{e} = -\frac{\mathrm{d}\beta}{\mathrm{d}\alpha} C_\mathrm{m} = -V C_\mathrm{m} = -T C_\mathrm{m} \tag{18}$$

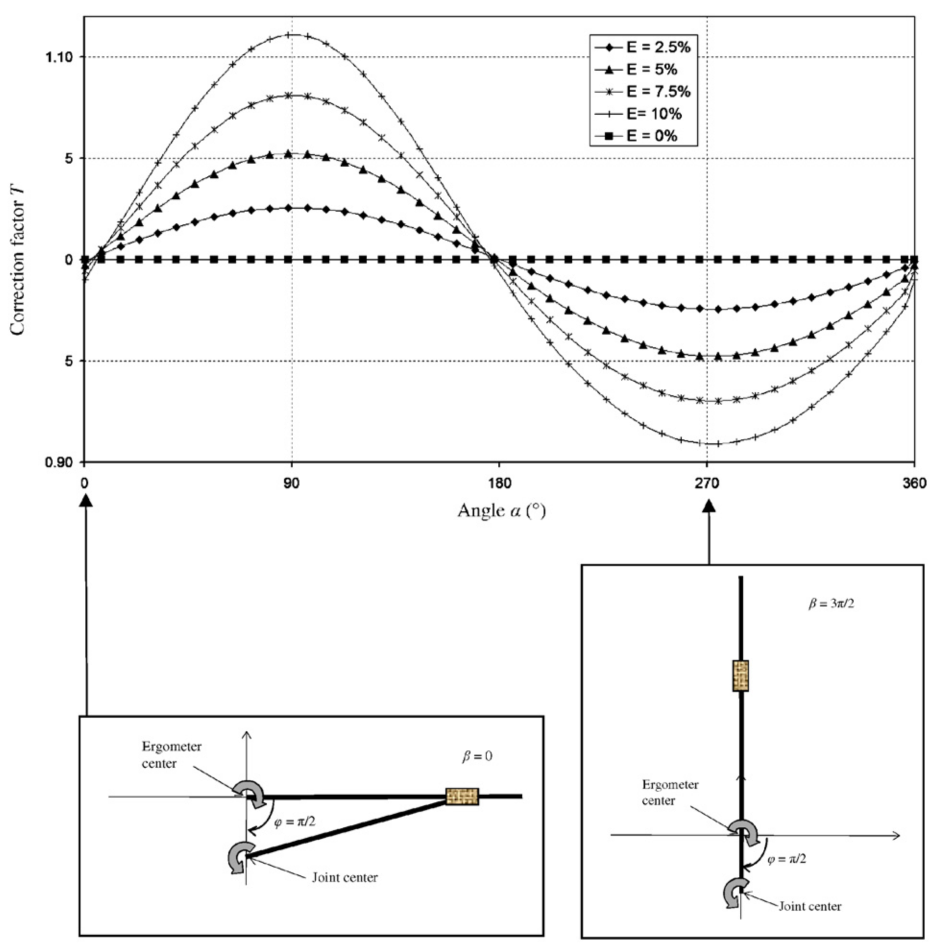

Fig. 4. Torque correction factor T dependence in α for $E(\%) = \{0, 2.5, 5, 7.5, 10\}$ and $\varphi = \pi/2$.

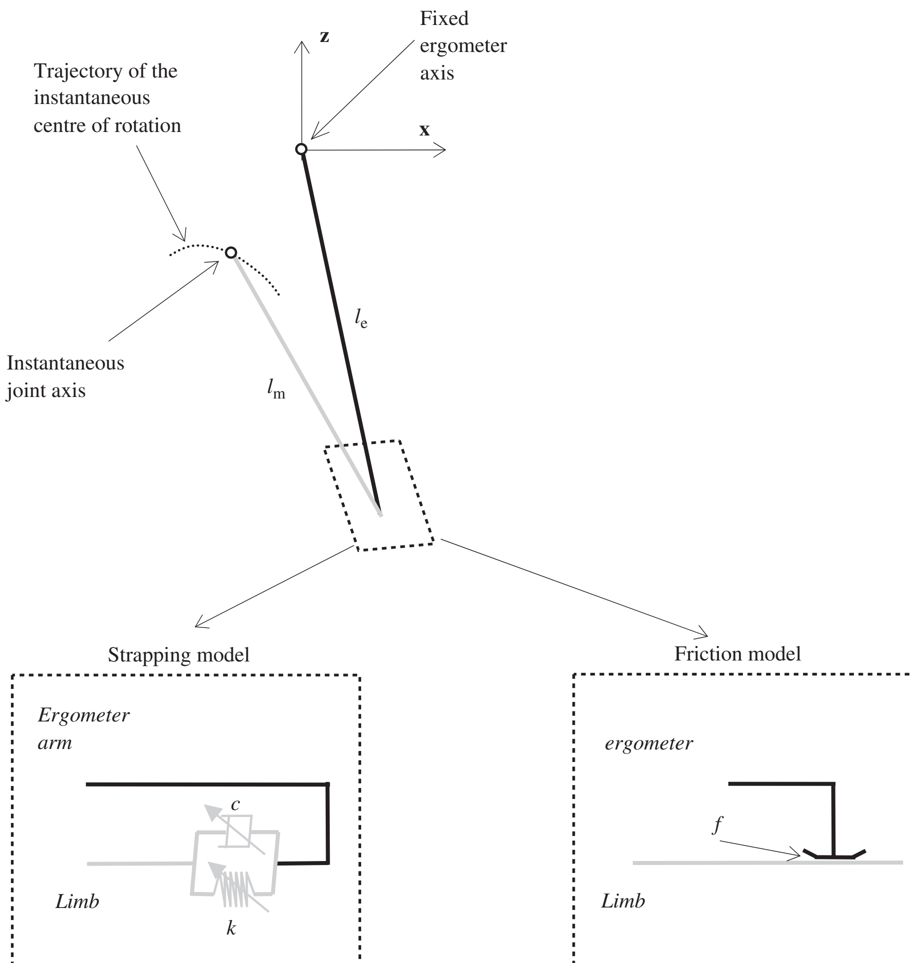

Fig. 5. Mechanical strapping: friction variables and parameters.

This means that the same corrections must be made when using the ergometer for isometrics.

2.2. Analytical model of strapping

The limb is now strapped to the ergometer arm and in the case of non-zero offset, a necessary relative movement is likely to occur unless the limb is damaged. This analysis is still restricted to the particular case of $\varphi = \pi/2$. To model the strapping, we propose stiffness associated to viscous dissipating terms as shown in Fig. 5 with stiffness acting along the limb, thus artificially creating a limb of variable length. The length of the limb l_m depends on the length l_e of the ergometer arm,

$$l_m = (l_e^2 + d^2 + 2dl_e \sin \alpha)^{1/2} = l_e(1 + E^2 + 2E \sin \alpha)^{1/2} \tag{19}$$

Since $E \ll 1$, a first-order Taylor expansion is worth considering,

$$l_m \simeq l_e(1 + E \sin \alpha) \tag{20}$$

Previous Lagrange equations have to be augmented with corresponding elastic potential energy E_e

$$E_e = \tfrac{1}{2}k(l_m - l_{m0})^2 \simeq \tfrac{1}{2}kE^2 l_e^2(\sin \alpha - \sin \alpha_0)^2 \tag{21}$$

where l_{m0} is the initial length for $\alpha = \alpha_0$.The dissipation function terms F_d are introduced as

$$F_d = \tfrac{1}{2}c\dot{l}_m^2 \simeq \tfrac{1}{2}cl_e^2 E^2 \dot{\alpha}^2 \cos^2 \alpha \tag{22}$$

Augmented Lagrange equations are now

$$\frac{d}{dt}\frac{\partial E_c}{\partial \dot{q}} - \frac{\partial E_c}{\partial q} = Q - \cancel{\frac{\partial E_p}{\partial q}} + A^t \lambda - \frac{\partial E_e}{\partial q} - \frac{\partial F_d}{\partial \dot{q}} \tag{23}$$

Thus, system (7) becomes

$$\begin{cases} 0 = C_e + V\lambda + C_s + C_v \\ 0 = C_m - \lambda \\ V\dot{\alpha} - \dot{\beta} = 0 \end{cases} \tag{24}$$

with

$$C_s = -\frac{\partial E_e}{\partial \alpha} \simeq -kE^2 l_e^2(\sin \alpha - \sin \alpha_0) \cos \alpha \tag{25}$$

and

$$C_v = -\frac{\partial F_d}{\partial \dot{\alpha}} \simeq -cl_e^2 E^2 \dot{\alpha} \cos^2 \alpha \tag{26}$$

Hence, the torque correction factor T now differs from V,

$$T = V + \frac{C_s + C_v}{C_m} \tag{27}$$

where stiffness and viscous terms may be considered as perturbation terms. In this section, first-order Taylor expansions are written for C_s and C_v but the numerical simulations take the exact expressions into account.

2.3. Computer model of friction

For some joints, no strapping is necessary and the limb is simply supported by the ergometer arm as shown in Fig. 5. Then, we introduce a dry friction (Coulomb) contact model in order to evaluate torque loss due to the friction. But Coulomb friction is not linear, which renders impracticable an analytic approach. Thus, we turn to a numerical model built using multi-body dynamics ADAMS software. First, analytic (offset + stiffness) and numerical models are compared through various simulations with different parameters. That identical results are found validates the analytical model; moreover, it gives reasonable confidence for future numerical results in particular for friction models.

3. Results

3.1. General

Simulations are performed with an $\alpha = 360^\circ$ amplitude starting with the limb in the horizontal position along the x-axis which corresponds to $\alpha_0 = 0$. The correction factor T is then plotted against α; a given angular position α_P of α is then defined associated to a range of motion (ROM). Simulations are realized for $E = 10\%$, $\varphi = \pi/2$, and the following torques 5, 25, 50 or 100 N m. For a zero offset, stiffness and friction have a null effect on correction factor T.

3.2. Offset of rotation centers

Fig. 4 shows the variation of corrector factor T due only to offset effects. It can be observed that the relative maximal errors for T are about the same order of magnitude as that of E (e.g. in %: $\Delta T/T = 3$ vs. $E = 2.5$; 5 vs. 5; 8 vs. 7.5; 11 vs. 10). Phase angle φ causes a shift of the curves as shown in Fig. 6 for $\varphi = 120^\circ$. Fig. 6 also shows the influence of α_P and ROM: for the same ROM, ROM1 = ROM2, $\Delta T/T$ varies from 8% for $\alpha_{P1} = 90^\circ$ to 11% for $\alpha_{P2} = 270^\circ$; similarly for $\alpha_{P2} = \alpha_{P3}$, $\Delta T/T$ varies from 8% for ROM2 to 11% for ROM3. It can also be remarked that the T curve is not symmetrical with respect to 1. This can easily be verified with a second-order Taylor expansion of T vs. E at maximum and minimum values leading to a shift of E^2,

$$T_{\max} = 1 + E + E^2 \tag{28}$$

$$T_{\min} = 1 - E + E^2 \tag{29}$$

Furthermore, it is worth noting that the correction factor is independent of the muscular torque. Using an intuitive approach, one might conclude that T is equal to length ratio, i.e. $T = l_e/l_m$. This is only true at the first order: $E = 10\%$ would lead to $T_{\max} = 1.10$ instead of $T_{\max} = 1.11$ with the upper model.

3.3. Stiffness and viscous effects

To evaluate stiffness, a dynamometer could be used to measure the effort elongation dependence in order to obtain the longitudinal limb stiffness. This is hardly practicable and consequently, it is necessary to propose a reasonable stiffness value intuitively. A 5 N effort for a 1 cm elongation seems appropriate. This leads to a 500 N m^{-1} stiffness. We also introduced a 1000 N m^{-1} stiffness in the simulation. In any case, stiffness behavior is not linear: the larger the amplitude, the higher the stiffness. For viscous friction constant c, we take $c = k/10$ corresponding to a time constant of 0.1 s in a first-order model approach.

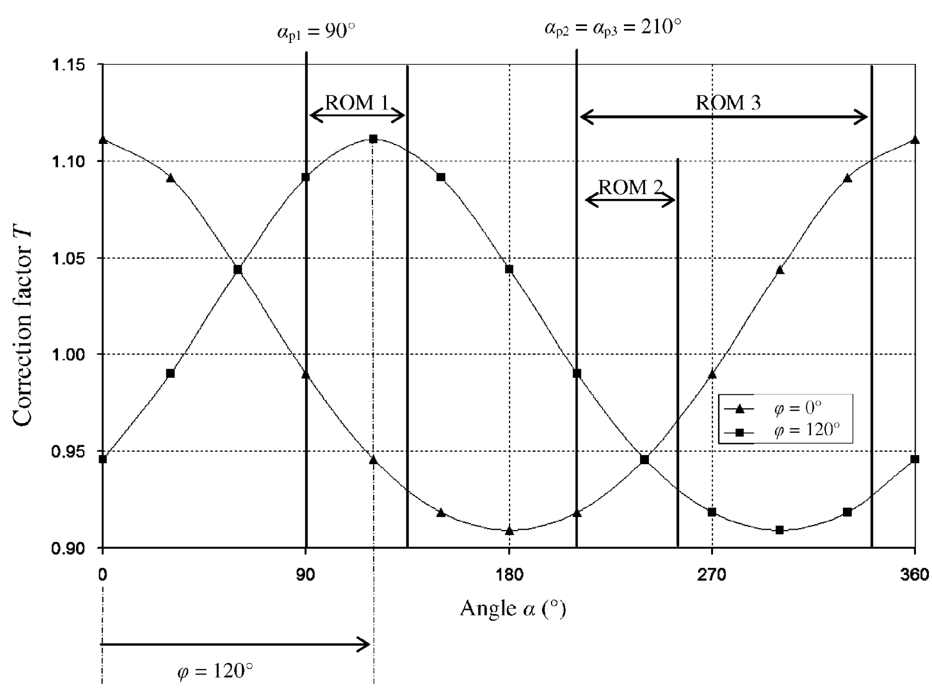

Fig. 6. Angle φ causes shift of the curves. Influence of α_P and ROM for $E = 5\%$.

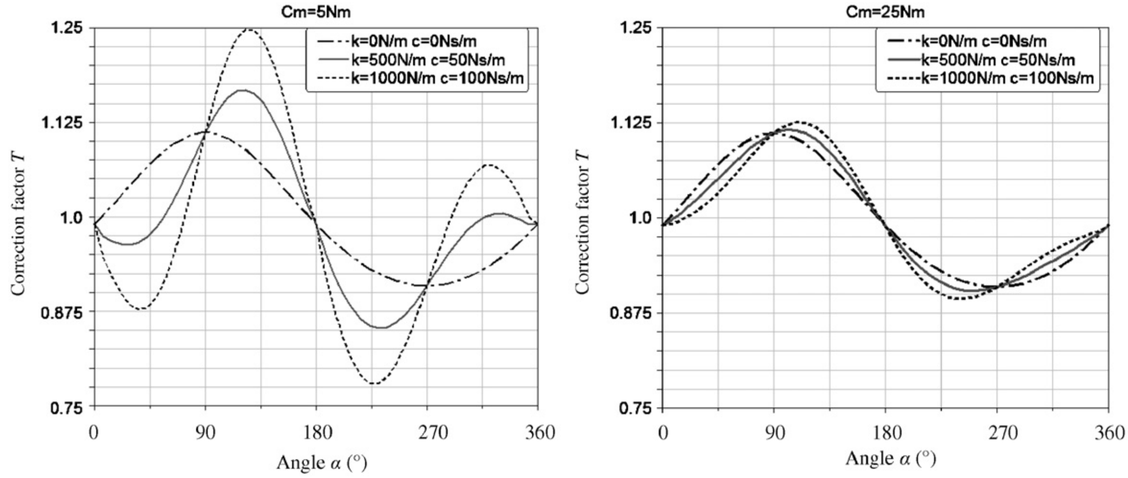

Fig. 7. Strapping model. Influence of stiffness and damping for $C_m = 5\,\mathrm{N\,m}$ and $C_m = 25\,\mathrm{N\,m}$.

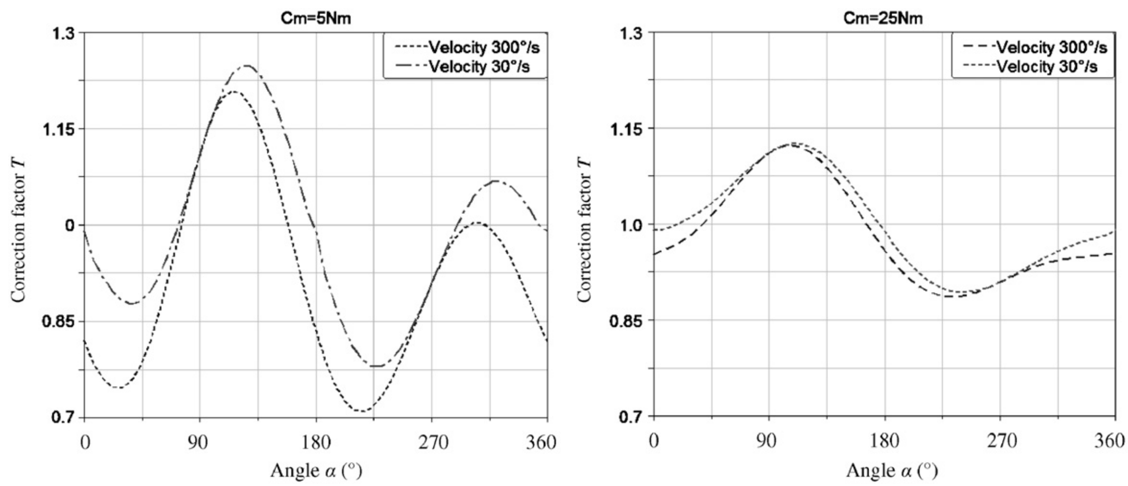

Fig. 8. Strapping model. Influence of velocity for $C_m = 5\,\mathrm{N\,m}$ and $C_m = 25\,\mathrm{N\,m}$.

Eq. (27) shows the higher the muscular torque C_m, the lower the correction factor stiffness contribution $(C_s + C_v)/C_m$. These results are shown in Fig. 7 for 5 and 25 N m torques. For 5 N m and $k = 1000\,\mathrm{N\,m^{-1}}$, maximum T jumps from 1.1 (only offset) to 1.25 (offset + stiffness); but with the same stiffness, stiffness influence decreases rapidly with respect to C_m and the strapping effects become negligible for torque higher than 25 N m. It should also be noted that the frequency variation stiffness of T has doubled in agreement with square cosine terms in Eq. (26).

The influence of viscous effects is shown in Fig. 8 for $k = 1000\,\mathrm{N\,m^{-1}}$, $c = 100\,\mathrm{N\,s\,m^{-1}}$, $30^\circ\,\mathrm{s^{-1}}$ and $300^\circ\,\mathrm{s^{-1}}$ and for 5 and 25 N m. It is observed that velocity effect decreases with increasing torque. In the case of $C_m = 25\,\mathrm{N\,m}$, T_{max} is found to be slightly larger than 1.11 which corresponds to only offset rotation centers.

3.4. Friction effects

For Coulomb friction between the limb and the ergometer arm, we selected a friction Coulomb coefficient $f = 0.5$, characteristic of tissue-strapping contact. Simulations were made for upper and lower values of f, 0.7 and 0.3, respectively.

Fig. 9 shows simulation results. We observe that the general shape of the curves is not affected very much. Nonetheless, if T_{max} is reached for all friction coefficient values, it is not the case for T_{min} since the higher the friction, the lower the value T_{min}.

4. Conclusion

For a zero offset of rotation centers, correction factor T remains constant and equal to 1: no corrections are then needed.

However, the correction factor, even in the presence of offset, may be close to 1 depending on the offset angular position angle φ and also on the range of motion. However, is it possible to check for a variable center of rotation? Of course, it becomes more complicated for joints with an instantaneous center of rotation: the knee or the neck for example (Sorensen et al., 1998; Rothstein et al.,

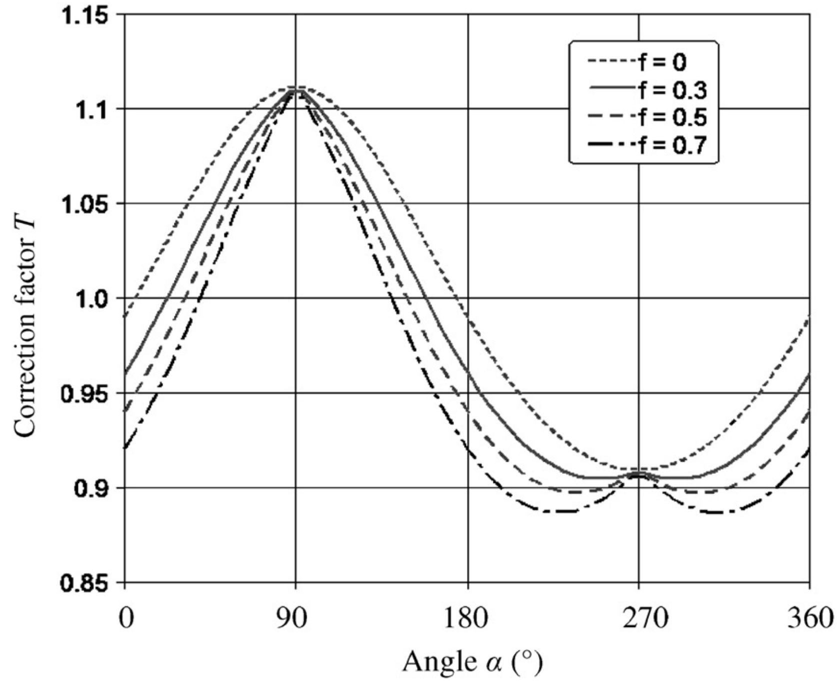

Fig. 9. Influence of friction.

1987). Again, this study is restricted to constant offset in magnitude and direction.

When offset occurs, gravity effects due to the limb mass are not completely corrected by the ergometer. As shown by the model, a residual torque still remains: it is then necessary to verify that this residual torque is negligible in comparison to the measured torque.

In the past, authors (Reimann et al., 1997) have already pointed out the offset rotation centers but, to our knowledge, nobody has underlined the combined effect of offset and strapping (stiffness and viscous effects) or offset and friction. This may be attributed to the satisfactory repeatability of the protocols and influential parameters (offset, stiffness, etc.). This strongly increases the inter-subject difference due to possible different inter-subject stiffnesses.

In the case of an isokinetic measurement with a strapped limb, it is shown that results are affected by stiffness effects only for low torque (less than 25 N m). Consequently, it is advisable to refer to Table 1, to be sure of torque dependence. If the test is done at a low muscular torque and at a high rotation velocity, viscous effects may be preponderant with respect to stiffness effects since they are directly proportional to velocity.

If the limb is not strapped, then friction effects influence the correction factor and, contrary to the preceding case, these effects are proportional to the applied torque. If extreme correction factor values of T are not affected very much, the shape of the curve is different.

For a position error to occur is hardly unavoidable, and this paper underlines the importance of the combined effect of offset and strapping or offset and friction between the limb and the ergometer arm. To overcome these unavoidable offset errors, a new mechanical system of torque measurement is needed. Such a system would be able to measure a torque for a variable center of rotation and also for general joint movement in space. Consequently, for accurate evaluations, ergometers equipped with a fixed axis are likely to be upgraded.

Conflict of interest

There are no conflicts of interest to be disclosed.

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